An Equivalent FIR Filter for an IIR Filter with Double Frequency Initialisation

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Indexing terms: *IIR, FIR, Transient Response, Initialization*

Abstract

IIR filters designed for steady state operation suffer from the transient effect caused by processing a finite number of samples. Their performance can be improved by using initialization techniques. This letter presents an equivalent FIR filter for an IIR filter operating in the transient and using double frequency initialization. This equivalent filter simplifies the transient analysis of IIR filters and provides a direct formula for calculating the transient frequency response and usable bandwidth.

Introduction

Digital infinite impulse response (IIR) filters are usually designed for steady state operation (i.e. processing a large number of input samples). If the number of processed samples is limited, such as in radar and sonar applications, then these filters will suffer from the transient effect. In this case, their frequency responses will be a function of the number of processed samples. These responses are called the transient frequency responses and they are different from the steady state responses.

One way of improving the transient frequency responses of IIR filters is by initializing their internal memories with values other than zeros. This requires an initialization processor to calculate the initial values from the first input sample in real time. These values will be the steady state values of the filter for a given input signal. The input signal may be approximated by a step function (step initialization) [1], or by a single tone sine wave (single frequency initialization) [2]. An initialization processor which can work at two frequencies (double frequency initialization) has also been introduced [3]. Other types of initialization for improving the transient performance of IIR filters exist [4, 5], however they are suitable for batch processing.

This letter derives an equivalent finite impulse response (FIR) filter for an IIR filter operating in the transient with double frequency initialization. The importance of this equivalent filter is that it simplifies the transient analysis of IIR filters with initialization by providing a direct formula for calculating the transient frequency response and usable bandwidth.

Double *frequency initialisation*

The transfer function of a real second order IIR notch filter is given by:

$$H(z) = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2}}, \qquad (1)$$

where $\{a_1, a_2\}$ are the feed forward and $\{b_1, b_2\}$ are the feedback real coefficients of the filter. This filter has two complex conjugate poles p_1 and $p_2 = p_1^*$, and two complex conjugate zeros on the unit circle $z_1 = \exp(j2\pi f_1 T_s)$ and $z_2 = \exp(j2\pi f_2 T_s) = z_1^*$, where f_1 is the notch frequency, $T_s = 1/f_s$ is the sampling period, f_s is the sampling frequency, and $f_2 = f_s - f_1$.

The z-transfer function of this filter can be rewritten as a product of two first order complex sections:

$$H(z) = \frac{1 - z_1 z^{-1}}{1 - p_1 z^{-1}} \times \frac{1 - z_2 z^{-1}}{1 - p_2 z^{-1}}. \qquad (2)$$

The initialization processor, shown in Figure 1, calculates the initial values of the internal memories for each section from the first received input sample $x(0)$ and feeds them to the associated section. The output of each section is fed to a multiplier so that the filter will have two zeros, at the initialized frequencies, in the transient frequency response irrespective of the number of processed samples. The initial values of the internal memories of the complex sections are calculated as follows [3]:

$$m_1 = \frac{x(0)\, z_1^{-1}}{1 - p_1 z_1^{-1}}, \quad m_2 = \frac{x(0)\, z_2^{-1}}{1 - p_2 z_2^{-1}}. \qquad (3)$$